

## Optimization problem

You need to dig and line an open channel with rectangular cross-section, width  $W$  and height  $H$ . The flow rate  $Q$  is given as  $n^{-1}AR^{2/3}S^{1/2}$  (Manning's equation);  $n$  is a dimensionless roughness coefficient,  $S$  is the channel slope,  $A = WH$  is the cross-sectional area, and  $R = A/P$  is the hydraulic radius where  $P = W + 2H$  is the wetted perimeter. The channel cost per unit length is given by  $C_E A + C_L P$ , where  $C_E$  is the cost rate for excavation and  $C_L$  for lining. Assuming that  $n = 0.03$ ,  $S = 0.0005$ ,  $C_E = \$300 \text{ m}^{-2}$ ,  $C_L = \$150 \text{ m}^{-1}$ ,  $Q = 1 \text{ m}^3/\text{s}$ , find the values of  $W$  and  $H$  that minimize total cost.