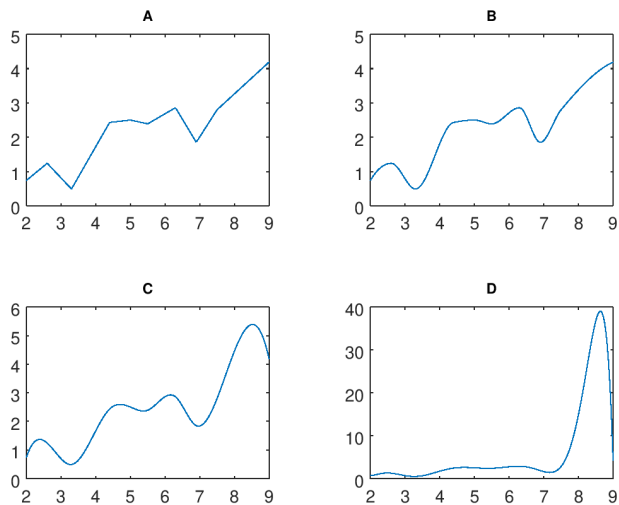


Practice final exam (9-12) problems

1

The subplots below show different interpolating functions fit to the same set of 10 points, which have x coordinates ranging from 2 to 9. Identify each function plotted and briefly explain how you know. The options are polynomial interpolation, linear spline, cubic spline, PCHIP, or none of those (only select none if you are sure that the other options are all impossible).



2

Write a Matlab script to draw a smooth-looking graph of a function that goes through the following points:

$$\begin{array}{l|cccccc} x & 0 & 1 & 3 & 4 & 6 \\ y & 7 & 8 & 5 & 3 & 2. \end{array}$$

3

Given the points

$$\begin{array}{l|ccccc} x_i & -2 & -1 & 0 & 1 & 2 \\ y_i & 16 & 6 & 4 & 4 & 0, \end{array}$$

estimate $y(x = 1.3)$ using (a) polynomial interpolation through all the given points, (b) polynomial interpolation through the three closest points, (c) piecewise linear interpolation.

4

Given the points

$$\begin{array}{l|ccccc} x_i & -2 & -1 & 0 & 1 & 2 \\ y_i & 1 & -2 & 1 & -2 & 1, \end{array}$$

estimate $y(x = 1.2)$ using (a) polynomial interpolation through all the given points, (b) polynomial interpolation through the three closest points, (c) piecewise linear interpolation.

5

(a) Given the (x, y) values $(1, 2), (2, 4), (3, 10), (5, 7)$ write in matrix form the linear system to solve for the b_i coefficients in the interpolating cubic spline with natural boundary conditions.

(b) If an interpolating cubic spline is as given below, find (1) what points it interpolates and (2) its value at $x = 0$.

$$\begin{aligned} & -1.27(x + 1)^3 + 6.54(x + 1) - 2 \text{ for } -1 \leq x \leq 1 \\ & 9.15(x - 1)^3 - 7.57(x - 1)^2 - 8.59(x - 1) + 2 \text{ for } 1 \leq x \leq 2 \\ & -6.63(x - 2)^3 + 19.89(x - 2)^2 + 3.74(x - 2) - 6 \text{ for } 2 \leq x \leq 3 \end{aligned}$$

6

Consider the problem of fitting a regression equation of the form $y = \beta_0 + \beta_1x + \beta_2x^2 + \beta_3xz$ to the following data:

$$\begin{array}{l|ccccc} x & 0 & 1 & 2 & 3 & 4 \\ z & 0.2 & 1 & 1.5 & 2 & 0.8 \\ y & 33 & 41 & 54 & 75 & 96 \end{array}$$

(a) Give the design matrix for the problem.

(b) Write code in Matlab to find the least-squares parameter values using the `\` operator.

7

You want to approximate the data below with the function $f(t) = c_1 + c_2e^t + c_3e^{-t}$.

$$\begin{array}{c|cccccc} t & 0 & 1 & 2 & 3 & 4 & 5 \\ y & 0 & 2 & 5 & 2 & 9 & 1 \end{array}$$

Find $\mathbf{A}^T\mathbf{A}$ and $\mathbf{A}^T\mathbf{b}$ for the system of normal equations $\mathbf{A}^T\mathbf{A}\mathbf{c} = \mathbf{A}^T\mathbf{b}$. You don't need to actually solve for the least-squares values of \mathbf{c} .

8

An air pollution sensor emits a signal (voltage, V) that depends on the pollutant concentration c (parts per million, ppm) following the relationship $c \approx \beta_1V + \beta_2V^2$. Given the calibration data below, use least squares regression to estimate β_1 and β_2 .

$$\begin{array}{c|cccccc} V & 0 & 1 & 2 & 3 & 4 & 5 \\ c \text{ (ppm)} & 0 & 2 & 4 & 6 & 10 & 12 \end{array}$$

9

Suppose that the data below are from an experiment on strength S of a concrete mix as a function of percent cement p .

$$\begin{array}{c|cccccc} \% \text{ cement} & 1 & 2 & 3 & 5 & 7 & 8 \\ \text{Strength (MPa)} & 1 & 2 & 3 & 7 & 11 & 12 \end{array}$$

Estimate the strength at 6% cement using the least-squares first-degree polynomial fit to the given data.

10

Suppose that the data below are from an experiment on strength S of a concrete mix as a function of curing time t .

$$\begin{array}{c|cccccc} \text{Time} & 0 & 1 & 2 & 4 & 8 & 16 & 32 \\ \text{Strength} & 5 & 6 & 6 & 8 & 12 & 21 & 58 \end{array}$$

(a) Write Matlab code to find the least-squares values for b_1, b_2 in the linear regression model

$$S(t) = b_1 + b_2e^{\sqrt{t}}.$$

(b) If instead we used the regression function

$$S(t) = \frac{10}{\beta_1 + e^{\beta_2(t-\beta_3)}},$$

and fitted parameter values $\beta_1 = 0.1, \beta_2 = -0.1, \beta_3 = 6$, find RSS, R^2 , adjusted R^2 , and AIC score.

11

Suppose that the data below are from an experiment on strain of a composite material as a function of time under load.

| | | | | | | | |
|--------|----|----|----|----|----|-----|-----|
| Time | 0 | 1 | 2 | 4 | 8 | 16 | 32 |
| Strain | 35 | 39 | 43 | 52 | 77 | 154 | 474 |

(a) Write Matlab code to find the least-squares values for b_1, b_2 in the linear regression model

$$S(t) = b_1 + b_2 e^{t/16}$$

(b) If instead these data were fitted with the regression function

$$S(t) = \frac{100}{\beta_1 + e^{\beta_2(t-\beta_3)}}$$

with $\beta_1 = 0.1, \beta_2 = -0.1, \beta_3 = 10$, find RSS, R^2 , adjusted R^2 , and AIC score.

12

Suppose a bridge's displacement due to vibration as a function of time follows the equation

$$y(t) = a_1 \sin(\omega t) + a_2 \cos(\omega t).$$

- (a) Estimate a_1 and a_2 using linear least squares fitting to the data below (and, e.g., solving the normal equations) if it is known that $\omega = 0.7$
- (b) Find RSS, R^2 , adjusted R^2 , and AIC score for this fit.
- (c) Based on your regression model, estimate $y(t = 7)$.
- (d) Write Matlab code to estimate ω along with a_1 and a_2 using nonlinear least squares fitting.

| | | | | | |
|-----|----|-----|-----|-----|-----|
| t | 1 | 2 | 3 | 4 | 5 |
| y | 71 | 160 | 175 | 107 | -11 |

13

If fluidity f of a concrete mix is supposed to follow temperature T as $\log(f) = b_0 + b_1 T$, estimate b_0 and b_1 using least squares linear regression (setting up and solving the system of normal equations) with the following measurements:

| | | | | | | |
|-----|--|----|----|-----|-----|------|
| T | | 10 | 20 | 30 | 40 | 50 |
| f | | 17 | 45 | 172 | 453 | 1572 |

14

An equation from mechanics is $M = A - e \sin(A)$. If M is 26 and e is 0.1, use 2 iterations of Newton's method from a reasonable initial guess to estimate A .

15

(a) Use 5 iterations of Newton's method to find a root of the function $f(x) = x^5 - 2x + 1$ starting with $x_0 = 0$. (b) Estimate the fractional error of your result.

16

Apply 3 iterations of the false position method to estimate where $x^3 - 3x^2 - x = -1$, for x between 0 and 1.

17

Given the system of equations

$$\begin{aligned}x^3 &= y^2 \\x + y &= 7,\end{aligned}$$

carry out an iteration of Newton's method to estimate x and y . Use the initial values $x_0 = 2, y_0 = 4$

18

Set up and do 1 iteration of Newton's method for finding an intersection point of the curves $x^2 + y^3 = 10$ and $y + y^2 = \sin(x)$, starting from $(x, y) = (1, 2)$.

19

Estimate where

$$f(x) = 2e^x - 3x$$

reaches a minimum, using 3 iterations of golden section search. Assume that the minimum lies between $x = 0.4$ and $x = 1$

20

The deflection of a beam under loading is described by $y(x) = x^5 - 2x^3 + x$. Use 4 iterations of golden section search with starting values $a = 0.1, b = 0.8$ to estimate for what x the deflection is maximum.

21

- (a) Estimate where $f(x) = -2e^{-x} - 2x^2$ reaches a maximum using 6 iterations of golden section search. Assume that the maximum lies between $x = 0.2$ and $x = 0.6$
- (b) Use 1 iteration of Newton's method to improve your estimate from (a).
- (c) Write Matlab code to solve this problem using `fminsearch`, with an initial guess of $x = 0.4$

22

- (a) Estimate the minimum of $f(x) = e^x - 2x$ using 5 iterations of golden section search assuming that it lies between $x = 0$ and $x = 2$.
- (b) Estimate the same minimum using 2 iterations of Newton's method.