Practice exam 3 (7-9) problems

1

Estimate $y(5)$ for the initial-value problem

$$\frac{dy}{dt} = \sqrt{y} - t + 1, \;\; y(1) = 3$$

using Euler's method (RK1) with step size $h = 1$.

2

(a) Use Euler's method with step size $h = 1/4$ to estimate $y(1)$ given $\frac{dy}{dt} = -2y, y(0) = 3$.
(b) Find the absolute error of this estimate compared to the analytic solution $y(t) = 3e^{-2t}$.
(c) Write Python code to solve for and display $y(1)$ using `solve_ivp`.

3

(a) Use Euler's method with a step size h of $1/3$ to estimate $y(t = 1)$ if $y(t = 0) = 0, dy/dt = y + 2\sqrt{t}$.
(b) Estimate $y(t = 1)$ for the same problem with the RK4 method and $h = 1$.
(c) Write Python code that uses `scipy.integrate.solve_ivp` to estimate $y(t = 1)$ and displays the estimated value.

4

Let $y(t)$ be the solution to $y' = 4te^{-y}$ satisfying $y(0) = 3$. Use Euler's Method with time step $h = 0.1$ to approximate $y(0.1), y(0.2), \ldots, y(0.5)$.

5

(a) Use Euler's method with a step size h of $1/3$ to estimate $y(t = 1)$ if $y(t = 0) = 0, dy/dt = y + 4t^2$.
(b) Estimate $y(t = 1)$ for the same problem with the RK4 method and $h = 1$.

6

Estimate $y(3)$ for the initial-value problem
$y' = \frac{\sqrt{y}}{2}, y(1) = 1$
using Heun's method (RK2) with step size $h = 1$.

7

Estimate $y(3)$ for the initial-value problem $y'' = \frac{\sqrt{y}}{2}, y(1) = 1, y'(1) = 0$ using Heun's method (RK2) with step size $h = 1$.

8

(a) Use 1 step of the RK4 method to solve for $y(x = 6)$ if $y(x = 0) = 2$ and $y' = x\sqrt{y}$.

(b) Write Python code for solving this problem and displaying the result.

9

Estimate $y(t = 2)$ if $y(t = 0) = 0, dy/dt = -2y + 3\sqrt{t}$
with the RK4 method and $h = 1$.

10

(a) Use the implicit Euler method with step size $h = 1/6$ to estimate $y(1)$ given $\frac{dy}{dt} = -2y, y(0) = 3$.
(b) Find the absolute error of the estimated $y(1)$ compared to the analytic solution $y(t) = 3e^{-2t}$.
(c) Write Python code to solve for and display $y(1)$.

11

Estimate $y(3)$ for the initial-value problem
$y'' = \frac{\sqrt{y}}{2}, y(1) = 5, y'(1) = 0$
using Heun's method (RK2) with step size $h = 1$.

12

For the mass-spring system with driving force as given below, use 2 equal-length steps of Euler's method to estimate $x(t)$ at $t = 1$. The initial conditions at $t = 0$ are $x = 1, x' = 0$.

$$x'' + x = 3\cos(t)$$

13

(a) Write a Python function, similar to the RK2 one we did in lab, to implement Euler's method for solving a system of first-order differential equations with given initial values. The first line should be
`def rk1(f,tspan,y0,n):`
(b) Write a Python script that uses this `rk1` and a step size h =0.01 to display $y(2)$ if $y'' = \sin(1/y)$ and $y(1) = 10, y'(1) = 3$.

14

Write a Python script that solves numerically the initial-value problem
$y'' + y' + y^2 = 1, y(0) = 2, y'(0) = -2$
and plot $y(t)$ for $0 \leq t \leq 10$.

15

Write a Python script that (a) numerically estimates $\Theta(2)$ if $\Theta'' = -9\Theta$ and $\Theta(0) = 1, \Theta'(0) = 0$ and (b) finds the absolute error of this estimate relative to the analytic solution $\Theta(t) = \cos(3t)$.

16

(a) Set up a system of linear equations (in matrix form) using finite difference with $n = 4$ to solve the equation $y'' + y' = x + 2$ subject to the boundary conditions $y(x = 0) = 0$ and $y(x = 3) = 1$.
(b) Solve this system using Gaussian elimination or LU decomposition, and sketch $y(x)$.

17

Set up a linear system in matrix form to solve the boundary-value problem $y'' + 5y' = 10$, $y(0) = 1$, $y(4) = 3$ for values of the function y(x), using second-order-accurate centered finite-difference approximations for the derivatives, with n=5. You do not need to solve the linear system.

18

Set up a system of algebraic equations (in matrix form) using finite difference with $n = 5$ for $y'' - y' + y = 2$ subject to the boundary conditions $y(0) = 0$ and $y'(3) = 2$. Use second-order-accurate (centered where possible) finite difference approximations. You only need to write down the system, not to solve it.

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Set up a system of algebraic equations (in matrix form) using finite difference with $n = 5$ to solve $y'' + y' - y = 4$ subject to the boundary conditions $y(x = 0) = 0$ and $y'(x = 3) = 2$. Use second-order-accurate (centered where possible) finite difference approximations. For full credit, solve this system and sketch $y(x)$.

20

Set up a system of algebraic equations (in matrix form) using finite difference with $n = 5$ to solve $y'' + y' + y = 2$ subject to the boundary conditions $y(x = 0) = 0$ and $y'(x = 1) = 2$. For full credit, solve this system and sketch $y(x)$.

21

(a) Set up a system of linear algebraic equations (in matrix form) using finite difference with $n = 5$ to solve $y'' + y = x + 4$ subject to the boundary conditions $y(x = 0) = 0$ and $y(x = 5) = 2$. Explain what the unknowns in this system represent.
(b) Write Python code to solve the same problem numerically and to plot $y(x)$.

22

(a) Set up a system of linear equations (in matrix form) using finite difference with $n = 4$ to solve the equation $y'' - y = x + 3$ subject to the boundary conditions $y(x = 0) = 0$ and $y(x = 2) = 0$.
(b) Solve this system (for example, using Gaussian elimination) and sketch $y(x)$.
(c) Write Python code to solve the boundary-value problem numerically and plot $y(x)$.

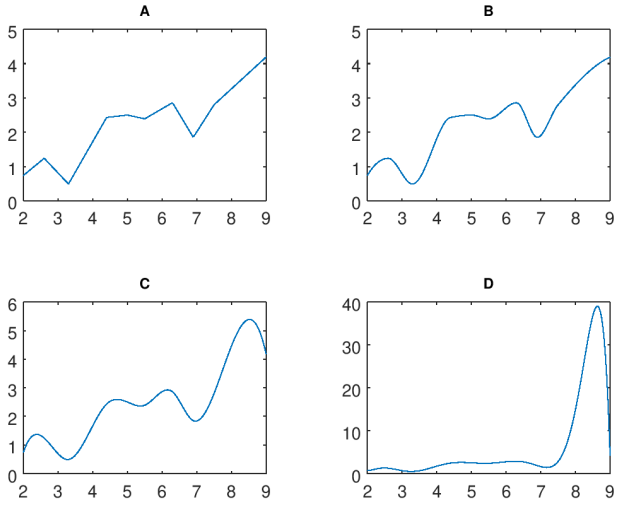
23

(a) Set up a system of linear equations (in matrix form) using finite difference with $n = 6$ to solve the equation $y'' + 3y' = y + 4$ subject to the boundary conditions $y(x = 0) = 0$ and $y(x = 1) = 0$.
(b) Solve this system (for example, using Gaussian elimination) and sketch the solution $y(x)$.

24

The subplots below show different interpolating functions fit to the same set of 10 points, which have x coordinates ranging from 2 to 9. Identify each

function plotted and briefly explain how you know. The options are polynomial interpolation, linear spline, cubic spline, PCHIP, or none of those (only select none if you are sure that the other options are all impossible).



25

Write a Python script to draw a smooth-looking graph of a function that goes through the following points:

x	0	1	3	4	6
y	7	8	5	3	2.

26

Given the points

x_i	-2	-1	0	1	2
y_i	16	6	4	4	0,

estimate $y(x = 1.3)$ using (a) polynomial interpolation through all the given points, (b) polynomial interpolation through the three closest points, (c) piecewise linear interpolation.

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Given the points

x_i	-2	-1	0	1	2
y_i	1	-2	1	-2	1,

estimate $y(x = 1.2)$ using (a) polynomial interpolation through all the given points, (b) polynomial interpolation through the three closest points, (c) piecewise linear interpolation.

28

(a) Given the (x, y) values $(1, 2), (2, 4), (3, 10), (5, 7)$ write in matrix form the linear system to solve for the b_i coefficients in the interpolating cubic spline with natural boundary conditions.

(b) If an interpolating cubic spline is as given below, find (1) what points it interpolates and (2) its value at $x = 0$.

$$-1.27(x+1)^3 + 6.54(x+1) - 2 \text{ for } -1 \leq x \leq 1$$
$$9.15(x-1)^3 - 7.57(x-1)^2 - 8.59(x-1) + 2 \text{ for } 1 \leq x \leq 2$$
$$-6.63(x-2)^3 + 19.89(x-2)^2 + 3.74(x-2) - 6 \text{ for } 2 \leq x \leq 3$$