Engineering Analysis Exam 1

Balance of forces for a damped linear oscillator gives

$$m\frac{d^2 x}{dt^2} = -c\frac{dx}{dt} - kx + h(t)$$

where c, k, m are positive numbers.

(a) Write this differential equation in standard form.

(b) Classify this differential equation. Be as specific as possible.

(c) Find the general solution for the case $h(t) = 0$.

(d) If $h(t) = 0$, $x(0) = b$, and $x'(0) = 0$, find the particular solution $x(t)$.

(e) For this particular solution, distinguish three regimes in the behavior of $x(t)$ at positive t, depending on the relative values of c, k, and m.

(f) If $h(t) = e^{-t}$, find a solution $x(t)$. Assume that e^{-t} is not a homogenous solution.

(g) Find the Laplace transform of the differential equation if $h(t) = a\delta(t)$, $x(0) = 0$, and $x'(0) = 0$. ($\delta(t)$ stands for the Dirac delta function.) Solve for $\bar{x}(s)$.

Possibly useful formulas:

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Integration by parts: $\int_a^b f(x) g'(x) dx = f(x) g(x)]_a^b - \int_a^b f'(x) g(x) dx$