Engineering Analysis Exam 2

1] (a) By assuming solutions in the form of power series about $x = 0$, find two independent solutions for the differential equation

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0 \quad .$$

(b) Determine the convergence radius of the two power-series solutions.
(c) This equation is a special case of what particular second-order ODE, discussed in class?

2] (a) Consider the proper Sturm-Liouville problem

$$\frac{d^2y}{dx^2}+\lambda y=0, \; y'(0)=y'(\pi)=0 \quad .$$

What are the characteristic numbers and characteristic functions?

(b) Use your results from (a) to solve the problem

$$\frac{d^2y}{dx^2}+3y=\cos(4x), \; y'(0)=y'(\pi)=0 \quad .$$

3] The function $f(x)$ is periodic with period 2π. Over $[-\pi,\pi]$, it is given by
$f(x)=1$ if x is in $[-\pi/2,\pi/2]$
$f(x)=0$ otherwise.
Find a Fourier series representation of $f(x)$, valid over the entire real line.

Possibly useful formulas:

Quadratic formula: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Integration by parts: $\int_a^b f(x)g'(x)dx = f(x)g(x)\big]_a^b - \int_a^b f'(x)g(x)\,dx$