Engineering Analysis Final Exam

1] Consider a rod with a uniform internal heat source that is kept at the background temperature at one end and insulated on the other, i.e.

$$\frac{\partial^2 T}{\partial x^2}+C=\frac{1}{\alpha^2}\frac{\partial T}{\partial t} \quad \text{for} \quad t>0, 0<x<L$$

where α^2 and C are positive constants, with boundary conditions

$$T(x=0,t)=0, \frac{\partial T}{\partial x}(x=L,t)=0, T(x,t=0)=f(x)$$ (arbitrary differentiable function).

(a) [10] Find the temperature distribution approached at large *t*. Sketch the graph of this distribution as a function of *x*, taking a system of units where *C* and *L* are both 1.
(b) [15] Obtain a PDE and boundary conditions for the transient component of the temperature distribution.
(c) [15] Solve for the transient temperature distribution. What is the timescale for the steady-state temperature distribution to be approached along the entire rod?
(d) [10] Solve for the temperature distribution if the heat source is not uniform, but given by $C(x) = x$.

2] Consider steady 2-dimensional heat conduction in the half-plane defined, in polar coordinates, by $0<\theta<\pi, r>1.$ Laplace's equation in polar coordinates is

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}=0.$$

The boundary conditions are

$$T(\theta=0,r)=0, T(\theta=\pi,r)=0, T(\theta,r=1)=g(\theta)$$ (arbitrary differentiable function).

Also, the temperature is bounded as $r\to\infty$.

(a) [10] Using separation of variables, decompose the PDE into ODEs in each of the two coordinates.
(b) [10] Solve the ODEs.
(c) [15] Find product solutions that satisfy the homogenous boundary conditions.
(d) [15] Find a superposition of these solutions that also satisfies the remaining boundary condition.